FULLY DEVELOPED TURBULENT FLOW WITH MACROVORTICES IN AN ANNULAR CHANNEL WITH ROTATING INNER CYLINDER

V. K. Shchukin

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Relations for computing the heat-transfer and friction coefficients for a fully developed turbulent flow with macrovortices are derived on the basis of an analysis of experimental heat-transfer data obtained for an annular channel with a rotating inner cylinder.

Experimental heat-transfer investigations for axial fluid flows through annular channels with a rotating inner cylinder [1, 2] showed that for constant values of the Reynolds number $\mathrm{Re} > 2000$, an increase in rotational speed at first does not affect the heat-transfer coefficient (turbulent state); after some time, the heat-transfer coefficient increases smoothly (turbulent state with macrovortices), and finally the relation $\mathrm{Nu} = f(\mathrm{Ta})$ becomes valid for any Reynolds number. The latter state, where heat transfer is determined solely by the rotational speed and is independent of the axial motion of the heat-transfer agent, will be termed the fully developed turbulent flow with macrovortices. The autonomy of the flow rate of the heat-transfer agent and of the heat-transfer intensity is an important characteristic of this state; it can be used in designing cooling systems and heat exchangers having rotating heat-transfer surfaces.

In the generalization of experimental heat-transfer data for an annular channel with a rotating inner cylinder, we will use a peripheral Reynolds number (Re_u) and the Taylor number (Ta). The first number describes the influence of the rotational speed on the velocity of the heat-transfer agent relative to the surface, while the second describes the influence of the mass forces on the flow [3]. These numbers are related by the simple expression

$$\mathrm{Ta} = \mathrm{Re}_u \sqrt{b/r_1}. \qquad (1)$$

Processing of experimental heat-transfer data for systems differing in their relative clearance [4–6] showed that for states where rotation has an effect on heat-transfer intensity, generalization of experimental data on the basis of the Taylor number makes it possible to obtain unique $\mathrm{Nu} = f(\mathrm{Ta})$ relations for a wide range of geometrical system parameters.

The results of experimental heat-transfer investigations for turbulent flow with macrovortices in a channel with a rotating inner cylinder without axial flow of the heat-transfer agent are generalized by the formula [6]

$$\mathrm{Nu}^* = 0.092\,(\mathrm{Ta}^2\mathrm{Pr})^{1/3}. \qquad (2)$$

This formula is obtained for $b/r_1 = 0.06–3.5$ and is recommended by the authors for $\mathrm{Ta} = 2 \cdot 10^4 – 6 \cdot 10^5$. Air, water, and methyl alcohol were used as the heat-transfer agent in the experiments.

In Fig. 1, the curve (a) corresponds to formula (2) and shows the experimental data on the basis of which the formula was obtained. Curve (b) characterizes the heat transfer in an annular channel without axial flow of the heat-transfer agent for laminar flow with macrovortices. It is plotted on the basis of the dimensionless equation [5]

$$\mathrm{Nu}^* = 0.42\,(\mathrm{Ta}^2\mathrm{Pr})^{1/4} \qquad (3)$$

for $\mathrm{Pr} = 1$.

From Figure 1, it may be seen that on the basis of the experimental data [6], relation (2) still holds for $\mathrm{Ta} = 2 \cdot 10^3$, thus confirming the possibility of obtaining both laminar flow and turbulent flow with macrovortices at Taylor numbers ranging from $2 \cdot 10^3 – 2 \cdot 10^4$. The onset of turbulence at $\mathrm{Ta} < 10^4$, observed in [6], may be attributed to the action of additional external effects on the flow during the experiments. Such effects might have occurred in the form of vibrations of the rotating cylinder.

Kosterin and Finat'ev have investigated the heat transfer in an annular channel with a rotating inner cylinder for the case of axial flow of the heat-transfer agent [1]. The experiments were performed with air at $b/r_1 = 0.271$ and $\mathrm{Re} = 2.5 \cdot 10^3 – 3 \cdot 10^4$. These experimental results, processed in the form of the relation $\mathrm{Nu}^*/\mathrm{Pr}^{1/3} = f(\mathrm{Ta})$, are also shown in Fig. 1. The experimental data for air were processed under the assumption that $\mathrm{Pr} = 0.7$. Moreover, because of the similar heat-transfer conditions at the surface of the inner and outer cylinders, the total heat-transfer coefficient was taken as half the heat-transfer coefficient at the inner surface.

The dashed curves in Fig. 1 correspond respectively to the $\mathrm{Nu}^*/\mathrm{Pr}^{1/3} = f(\mathrm{Ta})$ relations for turbulent flow and for turbulent flow with macrovortices, while the experimental points correspond to fully developed turbulent flow with macrovortices. From the figure, it can be seen that the heat transfer for fully developed flow with macrovortices is satisfactorily described by formula (2). Consequently, for fully developed turbulent flow with macrovortices, axial flow does not affect the heat-transfer coefficient, but rather constitutes the cause for the onset of turbulence in the flow and hence, defines the flow conditions of the fluid.

Because of the similarity of the heat-transfer conditions for turbulent flow with macrovortices in the absence of axial flow, and for fully developed turbulent flow in the presence of axial flow, one may assume that the dependence of the friction coefficient on the rotational speed is also similar for these flows. We shall derive this dependence on the basis of hydrodynamic heat-transfer theory.

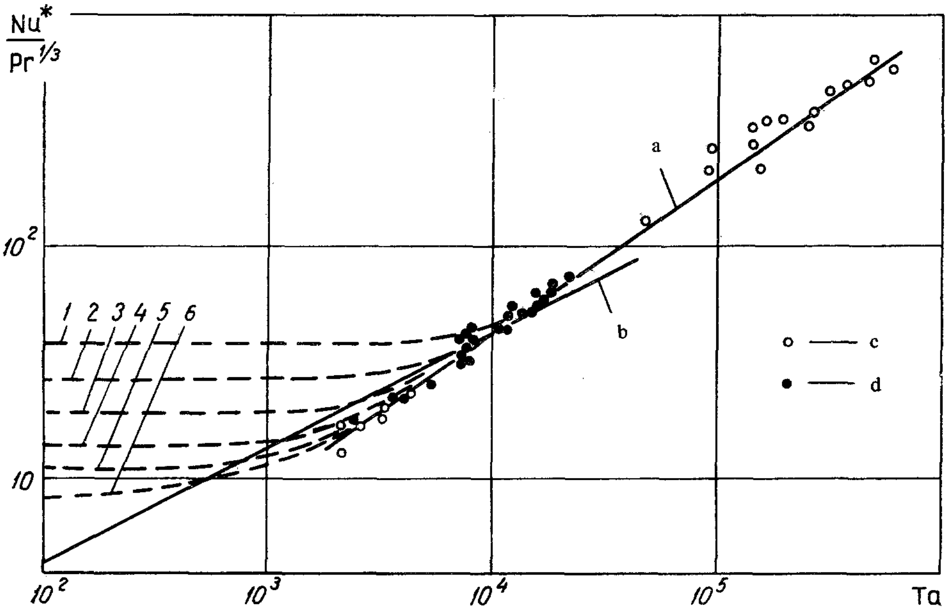

Fig. 1. Dependence of $Nu^*/Pr^{1/3}$ on the Taylor and Reynolds numbers with and without axial flow: (a) and (b) are respectively the turbulent and laminar flow with macrovortices without axial flow of the heat-transfer agent, (c) data from [6] for $b/r_1 = 0.06–3.5$, (d) data from [1] for $b/r_1 = 0.271$, (1) $Re = 3 \cdot 10^4$; (2) $Re = 2 \cdot 10^4$; (3) $Re = 1.45 \cdot 10^4$; (4) $Re = 1 \cdot 10^4$; (5) $Re = 3 \cdot 10^3$; (6) $Re = 2.5 \cdot 10^3$.

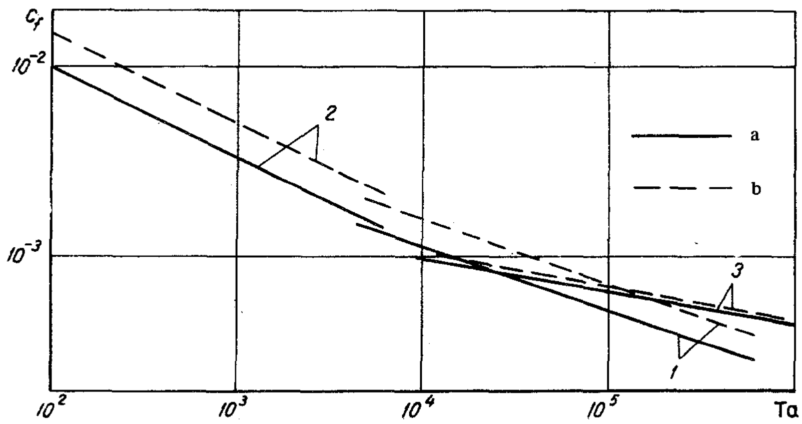

Fig. 2. Friction coefficient as a function of the Taylor number: (a) for $b/r_1 = 0.05$, (b) for $b/r_1 = 0.1$; (1) from formula (11), (2) from formula (7), (3) from formula (12).

Borisenko and Yantovskii [7] established a relation between Nu^* and the friction coefficient for an annular channel with a rotating inner cylinder with axial flow of the heat-transfer agent, under the assumption that $mPr = 1$ for air. Here, m is the ratio of the turbulent heat-exchange coefficient to the momentum-exchange coefficient, which, according to Prandtl's data, lies between 1.4 and 1.5 for a plane surface. The relation obtained,

$$Nu^* = c_f Re_u, \tag{4}$$

was used by the authors of [7] for deriving a formula to calculate the heat transfer in the clearance for laminar flow with macrovortices.

Because of the approximate nature of some of its assumptions, the hydrodynamic heat-transfer theory does not agree too well with the experiment. Experimental data concerning heat transfer and hydrodynamic resistance for laminar flow with macrovortices are now available which are well suited for assessing the accuracy of formula (4).

We shall introduce into the right-hand side of formula (4) a coefficient ε that takes into account the discrepancy between theory and experiment. With allowance for the physical characteristics of the fluid [5], and using the expressions (4) and (1) for laminar flow with macrovortices, we get

$$\varepsilon = \frac{Nu^*}{1.1\, c_f \, Ta \, Pr^{1/4}} \sqrt{\frac{b}{r_1}}. \tag{5}$$

In addition to formula (3), heat transfer for laminar flow with macrovortices is described by [2]

$$Nu^* = 0.409\, Ta^{*\,0.482}, \tag{6}$$

obtained for air at $Ta^* = 10^2–3.16 \cdot 10^3$ and $b/r_m = 0.054–0.246$.

Estimates of the hydraulic resistance under the conditions studied can be obtained with the aid of Wagner's formula

$$c_f = 0.76 \frac{(b/r_1)^{3/2}}{1 - 1/(1 + b/r_1)^2} Ta^{-0.478}, \tag{7}$$

which generalizes experimental data obtained for $Ta = 90–6 \cdot 10^3$ and $b/r_1 = 0.0278–0.1018$ [4], and Wendt's formula [8]:

$$c_f = 0.46\,(b/r_1)^{0.25}\, Re_u^{-0.5}, \tag{8}$$

obtained for $Re_u = 4 \cdot 10^2–10^4$.

By using formulas (3) and (8), one gets $\varepsilon = 0.83$, while formulas (6) and (7) lead to

$$\varepsilon = 0.5 \left[1 - \frac{1}{(1 + b/r_1)^2}\right] \frac{r_1}{b} Ta^{-0.022} \tag{9}$$

(for small relative clearances, where $Ta \cong Ta^*$.)

For $Ta = 10^2$ and $b/r_1 = 0.05–0.1$, formula (9) yields $\varepsilon = 0.837–0.79$, while for $Ta = 10^4$ it yields $\varepsilon = 0.775–0.717$.

Thus, for laminar flow with macrovortices, hydrodynamic theory developed on the basis of turbulent flow concepts correlates satisfactorily with the experiment at $\varepsilon \cong 0.77$. It should be noted that close correlation between the quantitative heat-transfer relations for laminar flow with macrovortices and turbulent flow in the absence of mass forces was observed in the investigation of other systems. This phenomenon is described for curvilinear channels in [9].

For turbulent flow with macrovortices, when the influence of the physical characteristics of the fluid is taken into account by the factor $Pr^{1/3}$ [6], formula (4) can be reduced to the form

$$Nu^* = 1.13\, \varepsilon\, c_f \, Ta \, Pr^{1/3} \sqrt{\frac{r_1}{b}}. \tag{10}$$

Having estimated Nu* from formula (2), we get

$$c_f = \frac{0.0817}{\varepsilon} Ta^{-1/3} \sqrt{\frac{b}{r_1}}. \tag{11}$$

For fully developed turbulent flow with macrovortices, this formula is applicable for $Ta = 2 \cdot 10^3–6 \cdot 10^5$, while for turbulent flow with macrovortices in the absence of axial flow, it is applicable for $Ta \geq 10^4$.

In Figure 2, the curves (1) are plotted from formula (11) for b/r_1 values of 0.05 (solid curve) and 0.1 (dashed curve), at $\varepsilon = 0.77$. For comparison, Fig. 1 includes the curves (2), plotted for laminar flow with macrovortices on the basis of formula (7) for the same values of b/r_1, and the curves (3), plotted from the theoretical formula [10]

$$\frac{1}{\sqrt{c_f}} = 5.464 + 3.536 \ln Re_u \sqrt{c_f} \tag{12}$$

for the friction coefficient in an annular channel with a rotating inner cylinder for turbulent flow. The formula was obtained without allowance for particle trajectory distortions or secondary flows.

It may be seen from Fig. 2 that formula (12) differs appreciably from (11) in regard to the nature of the dependence of c_f on Ta and b/r_1.

The conditions under which fully developed turbulent flow with macrovortices will occur can be estimated approximately on the basis of the experimental heat-transfer data obtained in [1]. From an analysis of experimental data obtained for $Re = 2.5 \cdot 10^3–3 \cdot 10^4$, it proved possible to obtain a formula for the critical Taylor number in the form:

$$Ta_{cr} = 133.5\, Re^{0.445}. \tag{13}$$

For $Ta \geq Ta_{cr}$, a fully developed turbulent flow with macrovortices will occur in an annular channel.

NOTATION

b is the clearance (spacing between the surfaces of two concentric cylinders); $c_f = \tau/(\rho u^2/2)$ is the friction coefficient; $F_g = f(b/r_m)$; $Nu^* = 2b\,\alpha^*/\lambda$ is the Nusselt number; Pr is the Prandtl number; q is the specific heat flux; r_1 is the radius of the inner cylinder; r_m is the mean radius of the annular channel;

$Re = 2bw/\nu$ is the Reynolds number for axial flow; $Re_u = bu/\nu$ is the peripheral Reynolds number; t_1 and t_2 are the surface temperatures of the inner and outer cylinder, respectively; t_f is the mean temperature of the fluid; $Ta = \omega r_1^{1/2} b^{3/2}/\nu$ is the Taylor number; $Ta^* = Ta/F_g$ is the modified Taylor number; u is the peripheral speed on the inner cylinder; w is the mean axial flow rate; $\alpha = q/(t_1 - t_f)$ is the heat-transfer coefficient; $\alpha^* = q/(t_1 - t_2)$ is the total heat-transfer coefficient; ε is the correction factor; λ is the coefficient of thermal conductivity of the fluid; ν is the kinematic viscosity factor; ρ is the density of the fluid; τ is the friction stress; ω is the angular velocity of the inner cylinder.

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